DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE
PERAMBALUR

DEPARTMENT OF SCIENCE AND HUMANITIES

QUESTION BANK

NUMERICAL METHODS-MA1251

UNIT-I

(SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS)

PART-A

1 .State the iterative formula for regula falsi method to solve f(x)=0

Solution:

The iteration formula to find a root of the equation f(x)=0

Which lies between x = a and x = b is

$X_1 = (a f(b) – b f(a)) / f(b) – f(a)$

2. How to reduce the number of iterations while finding the root of an equation by Regula falsi method.

Solution:

The number of iterations to get a good approximation to the real root can be reduced ,if we start with a smaller interval for the root.

3. State the order of convergence and convergence condition for Newton Raphson method.

Solution:

The order of convergence is 2

condition for convergence is $| f(x)f''(x) | < | f'(x)|^2$

4. Write the iterative formula of Newton Raphson method.

Solution:

$X_{n+1}= x_n – f(x_n) / f'(x_n)$

5. State the principle used in Gauss Jordan method.

Solution:

Coefficient matrix is transformed into diagonal matrix

6. For solving a linear system, compare Gaussian elimination method and Gauss Jordan method.

Solution:

Gaussian elimination method	**Gauss Jordan method**
Coefficient matrix is transformed into upper triangular matrix	Coefficient matrix is transformed into diagonal matrix
Direct method	Direct method
We obtain the solutions by back substitution method	No need of back substitution method

7. Write a sufficient condition for Gauss seidel method to converge

Solution: The Coefficient matrix should be diagonally dominant

8. Compare Gauss – Jacobi method and Gauss seidel methods

Solution:

Gauss-Jacobi method	**Gauss -Seidel method**
Convergence rate is slow	The rate of convergence of Gauss -Seidel method is roughly twice that of Gauss-jacobi.
Indirect method	Indirect method
Condition for convergence is the coefficient matrix is diagonally dominant	Condition for convergence is the coefficient matrix is diagonally dominant

9. Is the iteration method,a self - correcting method always?

Solution:

In general ,iteration is a self - correcting method, since the round off error is smaller.

10. Why Gauss -Seidel method is a better method than Jacobi's iterative method?

Solution:

Since the current value of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration ,the convergence in Gauss -Seidel method will be more rapid than in Gauss-Jacobi method.

11. Distinguish between direct and iterative (Indirect)method of solving Simultaneous equations.

Direct method	**Indirect method**
We get exact solution	Approximate solution
Simple, take less time	Time consuming laborious

12. Write the two methods to solve a system of equation

Solution:

(i) Direct method (ii) Indirect method

13. What is meant by diagonally dominant?

Solution:

A matrix is diagonally dominant if the numerical value of the leading Diagonal element in each row is greater than or equal to the sum of the numerical values of the other element in that row.

14. What is meant by self correcting method

Solution:

The error made in any computation is corrected in the subsequent iterations.

15.When Gaussian elimination method fails ?

Solution:

This method fails if the element in the top of the first column is zero. We can rectify this by interchanging the rows of the matrix.

PART-B

1. Find the positive root of $x^3 - 2x - 5 = 0$ by Regula falsi method.

2. Find an approximate root of $x \log_{10} x - 1.2 = 0$ by Regula falsi method

3. Solve $3x - \cos x = 1$ by Regula falsi method

4. Find by NR method ,the root of $\log_{10} x = 1.2$

5. By Newton Raphson find a non –zero root of $x^2 + 4 \sin x = 0$

6. Obtain Newton's iterative formula for finding N where N is a positive real number. Hence evaluate 142

7. Find the iterative formula for finding the value of 1 / N where N is a real number, using Newton Raphson method. Hence evaluate 1/26 correct to 4 decimal places

8. Find the negative root of $x^3 - \sin x + 1 = 0$ by using NR method

9. Solve the system of equations by Gauss elimination method

$$10x - 2y + 3z = 23$$
$$2x + 10y - 5z = -33$$
$$3x - 4y + 10z = 41$$

10. Solve the system of equations by Gauss Jordan method

$$2x + 3y - z = 5$$
$$4x + 4y - 5z = 3$$
$$2x - 3y + 2z = 2$$

11. Solve the system of equations by Gauss Jacobi method

$$2x - 3y + 20z = 25$$

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

12. Solve the system of equations by Gauss Jacobi method

$$2x - 3y + 20z = 25$$

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

13. Find the dominant eigen value and the corresponding eigen vector

Of the following matrix $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

14. Find all the eigen value of the following matrix

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

UNIT-II

(INTERPOLATION AND APPROXIMATION)

PART-A

1. **State Interpolation and Extrapolation.**

   Solution:

   The process of finding the value of a function inside the given range is called Interpolation.

   The process of finding the value of a function outside the given range is called Extrapolation.

2. **State Newton's forward interpolation formula.**

   Solution:

$$y_n = f(x_0 + nh)$$
$$= y_0 + ny_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + ...$$

3. **State Newton's backward interpolation formula.**

   Solution:

$$y_n = f(x_n + nh)$$
$$= y_n + ny_n + \frac{n(n+1)}{2!}\Delta^2 y_n + \frac{n(n+1)(n+2)}{3!}\Delta^3 y_n + ...$$

4. **State Gauss forward interpolation formula.**

   Solution:

$$y(x) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{n(n-1)(n+1)}{3!}\Delta^3 y_{-1} + \frac{n(n-1)(n-2)(n+1)}{4!}\Delta^4 y_{-2} + ...$$

5. **State Gauss backward interpolation formula.**

   Solution:

$$y(x) = y_0 + n\Delta y_{-1} + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{n(n-1)(n+1)}{3!}\Delta^3 y_{-2} + \frac{n(n-1)(n+1)(n+2)}{4!}\Delta^4 y_{-2} + ...$$

6. **State Newton's divided difference formula.**

   Solution:

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \dots.$$

7. **State Lagrange's Interpolation formula.**

   Solution:

   Let y = f(x) bea function such that f(x) takes the values $y_0, y_1, \dots y_n$ corresponding to $x_0, x_1, \dots x_n$

$$y = f(x)$$
$$= \frac{(x - x_1)(x - x_2)\dots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\dots(x_0 - x_n)} y_0 + \frac{(x - x_0)(x - x_2)\dots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\dots(x_1 - x_n)} y_1 + \dots.$$
$$+ \frac{(x - x_0)(x - x_1)\dots(x - x_{n-1})}{(x_n - x_1)(x_n - x_2)\dots(x_n - x_{n-1})} y_n$$

8. **State Inverse Interpolation formula.**

   Solution:

$$x = f(y)$$
$$= \frac{(y - y_1)(y - y_2)\dots(y - y_n)}{(y_0 - y_1)(y_0 - y_2)\dots(y_0 - y_n)} x_0 + \frac{(y - y_0)(y - y_2)\dots(y - y_n)}{(y_1 - y_0)(y_1 - y_2)\dots(y_1 - y_n)} x_1 + \dots.$$
$$+ \frac{(y - y_0)(y - y_1)\dots(y - y_{n-1})}{(y_n - y_1)(y_n - y_2)\dots(y_n - y_{n-1})} x_n$$

9. **When will you use Newton's backward interpolation formula.**

   Solution:

   We can apply the Newton's backward interpolation if the unknown value lies near the end of the table.

10. Using Lagrange's interpolation formula, find the polynomial for

x:	0	1	3	4
y:	-12	0	0	12

Solution:

$$y = f(x)$$

$$= \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)} y_1 + ...$$

$$+ \frac{(x-x_0)(x-x_1)...(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)...(x_n-x_{n-1})} y_n$$

$$= \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)} 0$$

$$+ \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} 0 + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}$$

$$= 2x^3 - 12x^2 + 22x - 12$$

11. Obtain the interpolation quadratic polynomial for the given data by using Newton's forward difference formula.

Solution:

The finite difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-3			
		8		
2	5		8	
		16		0
4	21		8	
		24		
6	45			

$$n = \frac{x-0}{2} = \frac{x}{2}$$

$$\therefore\ f(x) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2}\Delta^2 y_0$$

$$= x^2 + 2x - 3$$

12. Write down the range of n for which Gauss forward interpolation and Gauss backward interpolation gives accuracy result.

Solution:

For Gauss forward interpolation the range for n is 0 < n < 1 and for Gauss backward the range is -1 < n < 0.

13. Find the parabola of the form y = ax^2 + bx + c passing through the points (0,0) (1,1) and (2,20)

Solution:

By Lagrange's formula

$$y = \frac{(x-1)(x-2)}{(0-1)(0-2)}0 + \frac{(x-0)(x-2)}{(1-0)(1-2)}1 + \frac{(x-0)(x-1)}{(2-0)(2-1)}20$$

$$= 9x^2 - 8x$$

14. Write the Lagrange's fundamental polynomial L_0(x) and L_1(x) that satisfy the condition L_0(x) + L_1(x) =1 for the data [x_0 ,f(x_0)],[x_1,f(x_1)]

Solution:

$$L_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

15. State any two properties of divided difference.

Solution:

(i) The divided difference are symmetrical in all their arguments.

(ii) The divided difference of the sum or difference of two functions in equal to the sum or difference of the corresponding separate divided difference.

(iii)

16. What are the natural (or) free conditions in Cubic Spline.

Solution:

$S''(x_0() = 0, S''(x_n) = 0$ are called free conditions.

17. Define natural spline.

Solution:

If $S''(x_0() = 0, S''(x_n) = 0$ the cubic spline is called an natural spline.

18. State the properties of cubic spline.

Solution:

(i) $S(x_i) = y_i$, $i = 0,1,2,\ldots n$

(ii) $S(x), S'(x), S''(x)$ are continuous in [a,b].

(iii) S(x) is a cubic polynomial.

19. State the order of convergence of cubic spline.

Solution:

Order of convergence = 4.

20. State the error in N.F.I.F

Solution:

$$Error = \frac{u(u-1)(u-2)\ldots(u-n)h^{n+11} f^{n+1}(c)}{(n+1)!}$$

$$u = \frac{x - x_0}{n}$$

21. State the error in N.B.I.F

Solution:

$$Error = \frac{u(u+1)(u+2)...(u+n)h^{n+1}y^{n+1}(c)}{(n+1)!}$$

$$u = \frac{x - x_n}{n}$$

22. What is the assumption we make when Lagrange's formula is used?

Solution:

Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

23. What advantage has Lagrange's formula over Newton?

Solution:

The forward and backward interpolation formulae of Newton can be used only when the values of the independent variable x are equally spaced can also be used when the differences of the dependent variable y become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

24. What is disadvantage in practice in applying Lagrange's interpolation formula?

Solution:

Though Lagrange's formula is simple and easy to remember, its application is not speedy. It requires close attention to sign and there is always a chance of committing some error due to a number of positive and negative signs in the numerator and the denominator.

25. Show that $\underset{bcd}{\Delta}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$

Solution:

x	f(x)	Δ f(x)	Δ^2 f(x)	Δ^3 f(x)
a	$\frac{1}{a}$			
		$\frac{-1}{ab}$		
b	$\frac{1}{b}$		$\frac{1}{abc}$	
		$\frac{-1}{bc}$		$\frac{-1}{abcd}$
c	$\frac{1}{c}$		$\frac{1}{bcd}$	
		$\frac{-1}{cd}$		
d	$\frac{1}{d}$			

PART-B

1. **From the following data estimate the number of persons earning weekly wages between 60 and 70 rupees.**

Wages in Rs.	Below 40	40-60	60-80	80-100	100-120
No. of persons	250	120	100	70	50

2. **For the given values evaluate f(9) using Lagrange's formula.**

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

3. **Using Newton's divided difference formula find u(3) given u(1) = -26,**

   u(2) = 12, u(4) =256 and u(6) = 844.

4. **Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data.**

Year	1997	1999	2001	2003
Profit in lakhs of Rs.	43	65	159	248

5. **Using Newton's forward interpolation formula, find the polynomial f(x) satisfying the following data. Hence find f(2)**

x	0	5	10	15
f(x)	14	379	1444	3584

6. **Obtain the root of f(x) = 0 by Lagrange Inverse Interpolation given that**

f(30) = -30, f(34) = -13, f(38) = 3, f(42) = 18.

7. **Find f(x) as a polynomial in x for the following data by Newton's divided difference formula.**

x:	**-4**	**-1**	**0**	**2**	**5**
f(x):	**1245**	**33**	**5**	**9**	**1335**

8. **Find a polynomial of degree two for the data by Newton's forward difference method**

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
y	**1**	**2**	**4**	**7**	**11**	**16**	**22**	**29**

9. **Find f(8) by Newton's divided difference formula for the data:**

x:	**4**	**5**	**7**	**10**	**11**	**13**
f(x):	**48**	**100**	**294**	**900**	**1210**	**2028**

10. **Find the polynomial f(x) by using Lagrange's formula and hence find f(3) for**

x:	**0**	**1**	**2**	**5**
f(x):	**2**	**3**	**12**	**147**

11. **Find f(x) as a polynomial in x for the following data by Newton's divided difference formula.**

x:	**-4**	**-1**	**0**	**2**	**5**
f(x):	**1245**	**33**	**5**	**9**	**1335**

12. Using Newton's divided difference formula find f(x) and f(6) from the following data

x:	1	2	7	8
f(x):	1	5	5	4

13. From the following table, find the value of tan45º15´ by Newton's forward interpolation formula.

x º:	45	46	47	48	49	50
tan x º:	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

14. Fit the cubic spline for the data:

x:	0	1	2	3
f(x):	1	2	9	28

15. If f(0) = 0, f(1) = 0, f(2) = -12, f(4) = 0, f(5) = 600 and f(7) = 7308, find a polynomial that satisfies this data using Newton's divided difference interpolation formula. Hence, find f(6).

16. Given the following table, find f(2.5) using cubic spline functions:

i	0	1	2	3
x_i	1	2	3	7
$f(x_i)$	0.5	0.3333	0.25	0.2

17. The following values of x and y are given:

x:	**1**	**2**	**3**	**4**
y:	**1**	**2**	**5**	**11**

Find the cubic spline and evaluate y(1.5)

18. Use Lagrange's formula to fit a polynomial to the data:

x:	**-1**	**0**	**2**	**3**
y:	**-8**	**3**	**1**	**2**

and hence find y at x = 1

19. The following data are taken from the steam table:

Temp. ºC:	**140**	**150**	**160**	**170**	**180**
Pressure Kgf/cm^2	**3.685**	**4.854**	**6.302**	**8.076**	**10.225**

Find the pressure at temperature t = 142 º and t = 175 º.

UNIT-III

(NUMERICAL DIFFERENTIATION AND INTEGRATION)

PART-A

1. **What is the condition for Simpson's 3/8 th rule and state the formula?**

Solution:

The condition for Simpson's 3/8 th rule is that the number of intervals should be a multiple of 3.

$$\int_{x_0}^{x_0+nh} f(x)dx = 3h/8 \{(y_0+y_n)+3(y_1+y_2+y_4+y_5+\ldots.+y_{n-1})+2(y_3+y_6+y_9+\ldots y_n)\}.$$

2. **What is the condition for Simpson's 1/3 rd rule and state the formula?**

Solution:

The condition for Simpson's 3/8 th rule is that the number of intervals should be even number

$$\int_{x_0}^{X_n} f(x)dx = h/3\{(y_0+y_n)+2(y_2+y_4+\ldots)+4(y_1+y_3+\ldots)\}$$

3. **Write down trapezoidal formula.**

Solution:

$$\int_{x_o}^{x_n} f(x)\, dx = h/2\{(y_0+y_n)+2(y_1+y_2+y_3+\ldots y_{n-1})\}$$

4. **What is the order of error in the trapezoidal rule?**

Solution:

The error in the trapezoidal rule is of the order h^2

5. **What is the order of error in the Simpson's 1/3 rd rule ?**

Solution:

The error in the Simpson's 1/3 rd rule is of the order h^4

6. **What are the errors in the trapezoidal and Simpson's rules of numerical integration?**

Solution:

Error in trapezoidal rule = $-(b-a)/12\{h^2y''(x)\}$

Error in Simpson's rule = $-(b-a)/180\{h^4y^{iv}(x)\}$

7. **When can numerical differentiation be used?**

Solution:

When the function is given in the form of table of values instead of giving analytical expression we use numerical differentiation.

8. **Why Simpson's one-third rule is called a closed formula?**

Solution:

Since the end points y_0 and y_n are included in the simpson's $1/3^{rd}$ rule it is called closed formula.

9. **Write the formula for $\frac{dy}{dx}$ at $x \neq x_n$ using backward difference operator.**

Solution:

$$\left(\frac{dy}{dx}\right)_{x \neq xn} = \frac{1}{h}\{\nabla y_n + \frac{2v+1}{2}\nabla^2 y_n + \frac{3v2+6v+2}{6}\nabla^3 y_n + \ldots\}$$

10. **Write the formula for $\frac{dy}{dx}$ at $x=x_0$ using forward difference operator**

Solution:

$$\left(\frac{dy}{dx}\right)_{x \neq x0} = \frac{1}{h}\{\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 - \ldots\}$$

11.**Find $\frac{dy}{dx}$ at x = 1 from the following table**

x	1	2	3	4
y	1	8	27	64

Solution:

The forward difference table is as follows.

x	y	Δ	Δ^2	Δ^3
1	1			
		7		
2	8		12	
		19		6
3	27		1	
		37		
4	64			

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2}{2} y_0 + \frac{\Delta^3}{3} y_0 \ldots\right) \quad \text{-\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

Here h=1,x_0=1, Δy_0=7, $\Delta^2 y_0 = 12, \Delta^3 y_0 = 6$

-------(1)$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{1}\left(7 - \frac{12}{2} + \frac{6}{3}\right) = 3$

12. **Using Newton's backward difference formula, write the formulae for the first and second order derivatives at the end values $x = x_n$ upto the fourth order difference term.**

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + ...\right]$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + ...\right]$$

$$\left(\frac{d^3 y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + ...\right]$$

13. **When does Simpson's rule give exact result?**

Solution:

Simpson's rule will give exact result, if the entire curve $y = f(x)$ is itself a parabola.

14. **Six sets of values of x and y are given (x's being equally spaced), write the formula to get $\int_{x_1}^{x_6} ydx$**

Solution:

$$\int_{x_1}^{x_6} ydx = \frac{h}{2}[(y_1 + y_6) + 2(y_2 + y_3 + y_4 + y_5)]$$

15**. What are the errors in Trapezoidal and Simpson's rule of numerical integration?**

Solution:

Error in Trapezoidal rule $|E| < \frac{(b-a)}{12}h^2$. M in the interval $(a,b), h = \frac{b-a}{n}$

Error in Simpson's rule $|E| < \frac{(b-a)}{180}h^4 M$

16. **In order to evaluate** $\int_{x_0}^{x_n} y dx$ **by Simpson's 1/3 rule as well as by Simpson's 3/8 rule, what is the restriction on the number of intervals?**

Solution:

Let n = intervals

Rule: Simpson's 1/3 rule = The number off ordinates is odd (or)

The intervals number is even.

Simpson's 3/8 rule = n is a multiple of 3.

17.**Using Trapezoidal rule evaluate** $\int_0^{\Pi} \sin x dx$ **by dividing the range into 6 equal parts.**

Solution:

$$\int_0^{\Pi} \sin x dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

$$= \frac{\Pi/6}{2}\left[(0+0) + 2(0.5 + 0.866 + 1 + 0.866 + 0.5)\right]$$

$$= \frac{\Pi}{12}[7.464]$$

$$= 0.622$$

18. **Write down the trapezoidal rule to evaluate** $\int_1^6 f(x)dx$ **with h = 0.5**

Solution:

Here f(x) = h = 0.5

x	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
y=f(x)	f(1)	f(2)	f(3)	f(4)	f(5)	f(6)	f(7)	f(8)	f(9)	f(10)	f(11)

By formula, $\int_1^6 f(x)dx = \frac{h}{2}\left[(\textit{sum of the first and last ordinate}) + 2(\textit{remaining ordinate})\right]$

$$\frac{h}{2}[f(1) + f(11)] + 2[f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)]$$

PART B

1. **Evaluate the integral $\int_{1}^{2} dx / 1+ x^2$ using Trapezoidal rule with two sub intervals.**

2. **Dividing the range into 10 equal parts, fine the value of $\int_{0}^{\pi/2} \sin x \, dx$ by**

   i) Trapezoidal rule ii)Simpson's rule.

3. **By dividing the range into ten equal parts, evaluate $\int_{0}^{\Pi} \sin x \, dx$ by**

   Trapezoidal rule and Simpson's rule.Verify your answer with integration.

4. **Evaluate $\int_{0}^{6} (dx /1+x^2)$ by i) Trapezoidal rule ii)Simpson's rule. Also check up**

   the results by actual integration .

5. **Evaluate $\int_{0}^{5} (dx / 4x+5)$ by Simpson's one – third rule and hence find the value of**

   $\text{Log}_e 5$ (n= 10).

6. **Evaluate $\int_{0}^{2} (dx/ x^2 +4)$ using Romberg's method . Hence obtain an approximate**

   Value for π.

7. **Using Romberg's method evaluate $\int_{0}^{1} (dx/1+x)$ correct to three places of decimals.**

8. Using three – point Gaussian quadrature formula, evaluate

i) $\int_{-1}^{1} (1/ 1+x^2)\, dx$

ii) $\int_{0}^{1} (1 / 1+t^2)\, dt$

9.Evaluate $\int_{0.2}^{1.5} e^{-x^2}\, dx$ **using the three point Gaussian quadrature.**

10. Evaluate $\int_{0}^{2} \int_{0}^{2} f(x, y)\, dx\, dy$ **by Trapezoidal Rule for the following data :**

y/ x	0	0.5	1	1.5	2
0	2	3	4	5	5
1	3	4	6	9	11
2	4	6	8	11	14

11. Using Simpson's 1/ 3 rule evaluate $\int_{0}^{1} \int_{0}^{1} (1 / 1+ x+ y)\, dx\, dy$ **taking h =k = 0.5**

12. Evaluate $\int_{1}^{2} \int_{1}^{2} (dx\, dy/ x^2 + y^2)$ **numerically with h= 0.2 along x- direction and k= 0.25 along y- direction.**

13. Compute $\int_{4}^{5.2} \log_e x \, dx$ using Simpson's 1/ 3 and 3/8 rule.

14. Evaluate $\int_{-3}^{3} x^4 \, dx$ using (i)Trapezoidal rule (ii)Simpson's rule. Verify your results by actual integration.

15. The velocity v of a particle "s" from a point on its path is given by the table. Estimate the time taken to travel 60 feet by using Simpson's 1/ 3 rule

S(m)	0	10	20	30	40	50	60
V(m/s)	47	58	64	65	61	52	38

16. The table given below reveals the velocity v of a body during the time "t" specided. Find its acceleration at t = 1.1

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.1	60.8

17. Find the first and second derivate of the function tabulated below at x = 0.6

x	0.4	0.5	0.6	0.7	0.8
y	1.5836	1.7974	2.0442	2.3275	2.6511

18.Evaluate $\int_{0}^{1} (dx/1+x)$ by two point Gaussian quadrature.

UNIT-IV

(INITIAL VALUE PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS)

PART-A

1. **State the disadvantage of Taylor series method.**

In the differential equation $\frac{dy}{dx}$ =f(x,y), the function f(x,y) may have a

complicated algebraical structure. Then the evaluation lf higher order derivatives may become tedious. This is the demerit of the method.

2. **Write down the fourth order Taylors Algorithm**.

$$Y_{m+1} = y_m + hy_m' + \frac{h^2}{2} y_m'' + \frac{h^3}{3} y_m''' + \frac{h^4}{4} y_m''''$$

Here $y^n{}_m$ denotes the r^{th} derivatives of y w.r.to x at the point (x_m ,y_m).

3. **Which is better Taylor's method or R.K. Method?**

R.K. methods do not require prior calculation of higher derivatives of y(x), as the Taylor method does. Since the differential equations using in applications are often complicated, the calculation of derivatives may be difficult.

Also the R.K. formulas involve the computation of f(x,y) at various positions, instead of derivatives and this function occurs in the given equation.

4. **State modified algorithm to solve $y' = f(x,y)$, $y(x_0)= y_0$ at $x=x_0+h$.**

$$Y_{n+1}=y_n+hf\left(x_n+\frac{h}{2}, y_n+\frac{h}{2} f(x_n,y_n)\right)$$

$$Y_1=y_0+hf\left(x_0+\frac{h}{2}, y_0+\frac{h}{2} f(x_0,y_0)\right)$$

5. **Write down the Runge-Kutta formula of fourth order to solve $\frac{dy}{dx}=f(x,y)$ with $y(x_0)=y_0$.**

Let h denotes the interval between equidistant values of x. If the initial values are (x_0,y_0), the first increment in 'y' is computed from the formulas

$$K_1 = hf(x_0,y_0)$$

$$K_2 = hf\left(x_0+\frac{h}{2},y_0+\frac{k_1}{2}\right)$$

$$K_3 = hf\left(x_0+ \frac{h}{2},y_0+\frac{k_2}{2}\right)$$

$$K_4 = hf(x_0+ h,y_0+k_3)$$

$$\text{And } \Delta y=\frac{1}{6} (k_1+k_2+2k_3+k_4)$$

$$\text{Then } x_1=x_0+h,\ y_1=y_0+\Delta y$$

The increment in y in the second interval is computed in a similar manner

Using the same four formulas, using the values x_1,y_1 in the place of x_0,y_0 respectively.

5. **State the special advantage of Runge-Kutta method over Taylor series method.**

Runge-Kutta methods do not require prior calculation of higher derivatives of y(x), as the Taylor method does. Since the differential equations using in applications are often complicated, the calculation of derivatives may be difficult.

Also the Runge-Kutta formulas involve the computation of f(x,y) at various positions, instead of derivatives and this function occurs in the given equation.

6. **Write down Adams-Bashforth predictor formula.**

Adam's predictor corrector formulas are

$$Y_{k+1,p} = y_k + \frac{h}{24}(55y_k' - 59y_{k-1}' + 37y_{k-2}' - 9y_{k-3}')$$

$$Y_{k+1,c} = y_k + \frac{h}{24}(9y_{k+1}' + 19y_k' - 5y_{k-1}' + 9y_{k-2}')$$

7. **How many prior values are required to predict the next value in Adam's method?**

Four prior values.

8. **What is a Predictor-Corrector method of solving a differential equation?**

Predictor-Corrector methods are methods which require the values of y at $x_n, x_{n-1}, x_{n-2}, \ldots\ldots$ for computing the value of y at x_{n+1}.

We first use a formula to find the value of y at x_{n+1} and this is known as a predictor formula. The value of y so got is improved or corrected by another formula known as corrector formula.

PART-B

1. Using Taylor series method find y at x=0.1 if $\frac{dy}{dx}=x^2y-1$,Y(0)=1.

2.Solve y' = x + y ; y(0)=1 by Taylors series method. Find the values y at x = 0.1 and 0.2.

3.Solve $\frac{dy}{dx} = y_2 + x_2$ with y (0) =1. Use Taylor Series at x = 0.2 and 0.4. Find x = 0.1.

4. i) Using Taylor Series method find y at x = 0.1correct to four decimal places from $\frac{dy}{dx} = x^2 - y$, y (0) = 1, with h= 0.1. Compute terms upto x_4.

ii) Using Taylor's Series method , find y (1.1) given y' = x+y, y (1) = 0.

5. Using Taylor series method with the first five terms in the expansion find y (0.1) correct to three decimal places, given that $\frac{dy}{dx} = e^x - y^2$, y(0) ==1.

6. i)By Taylor's series method find y (0.1) given that y '' = y+ xy' ,y(0) = 1, y' (0) =0.

ii)Using Taylor's series method find y (1.1) and y (1.2) correct to four decimal places given y' = $xy^{1/3}$ and y(1) =1.

7. Using modified Euler's method solve; given that y'= 1-y, y (0) = 0 find y(0.1), y(0.2) and y (0.3).

8.Solve $\frac{dy}{dx}$ =$\log_{10}$(x+y), y(0) =2 by Euler's modified methgod and find the values of

y (.2), y (.4), and y (.6) by taking h=0.2

9. i) Using modified Euler's method solve; given that y' = y-x^2+1, y(0) = 0.5 find y (0.2).

ii) Using modified Euler's method; find y(0.1), y (0.2); given

$\frac{dy}{dx}$ = x^2+y^2, y(0) =1.

10. Compute y (0.2)and y (0.4) from y' = $\frac{y^2 - x^2}{y^2 + x^2}$ given that y (0)=1

11. Apply Runge – Kutta method to find approximate value of y for x =0.2 in steps of 0.1 if $\frac{dy}{dx}$ = x +y^2 given that y=1 when x =0.

12.Compute y (0.2) ,y (0.4) and y (0.6) ,given y' =x^3 + y, y (0) =2 by using Runge – Kutta fourth order method.

13.Given y ''- 2y' + 2y = e^{2x} sinx with y (0) = -0.4 and y' (0) = 0.6, using fourth order Runge – Kutta method find y(0.2).

14. Solve y' = 1/2 (1+ x^2)y^2, y (0) =1, y (0.1) = 1.06, y (0.2) = 1.12, y(0.3) = 1.21 compute y (0.4), using Milne's predictor corrector formula.

15. Solve 5xy'+y^2 =2, y (4) = 1, y (4.1) = 1.0049, y (4.2) = 1.0097, y (4.3) =1.0143, compute y (4.4) Milne's methods.

16. **Given y' = xy + y^2, y (0) = 1.0, find y (0.1) by Taylors method y (0.2) by Euler's method y (0.3) by Runge – Kutta and y (0.4) by Milne's method.**

17. **Solve y' = x-y^2, $0 \le x \le 1$, y(0) = 0, y(0.2) =0.02, y (0.4) =0.0795, y(0.6) = 0.1762, by Milne's method to find y (0.8)and y (1).**

18. **Compute the first 3 steps of the initial value problem** $\frac{dy}{dx} = \frac{x-y}{2}$

y (0) = 1.0 by Taylor series method and next step by Milen's method with step length

h =0.1.

19. **Given** $\frac{dy}{dx}$ **= x^3 + y, y (0) =2, y(0.2) =2.073, y (0.4) = 2.452, y (0.6) = 3.023**

compute y (0.8) by Milen's predictor – corrector method by h =0.2.

20. **Given** $\frac{dy}{dx}$ **= x^2 (1+y), y (1)= 1, y(1.1) = 1.233, y (1.2) = 1.548,y (1.3) = 1.979,**

evaluate y (1.4) by Adam's Bashforth method.

21. **Consider dy /dx = y- x2 +1, y (0) =0.5**

   i) using the modified Euler method find y (0.2)

   ii) using R-K method fourth order method find y (0.4) and y (0.6)

   iii)using Adam's Bashforth predictor- corrector method find y (0.8)

22. **Evaluate y (1.4): given y' = 1/x2 – y/x, y (1) =1, y (1.1) = 0.996, y (1.2) =0.986, y (1.3) = 0.972 by Adam's Bashforth formula.**

UNIT-V

(BOUNDARY VALUE PROBLEM IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS)

PART-A

1. **State the conditions for the equation . $Au_{xx}+Bu_{yy}+Cu_{yy}+Du_x+Eu_y+Fu=G$ Where A,B,C,D,E,F,G are function of x and y to be (i) elliptic (ii) parabolic(iii)hyperbolic**

   Solution: The given equation is said to be

   (i) Eliptic at a point (x,y) in the plane if $B^2-4AC<0$

   (ii) Parabolic if $B^2-4AC=0$

   (iii) Hyperbolic if $B^2-4AC>0$

2. **State the condition for the equation $Au_{xx}+2Bu_{xy}+Cu_{yy}=f(u_x,u_y,x,y)$ to be**

   (i) elliptic (ii) parabolic(iii)hyperbolic when A,B,C are functions of x and y

solution:

The equation is elliptic if $(2B^2)-4AC<0$

i.e. $B^2-AC<0$. It is parabolic if $B^2-AC=0$ and hyperbolic if $B^2-AC>0$

3. **What is the classification of $f_x-f_{yy}=0$**

   Solution:

   Here A=0 ,B=0,C=-1

   $B^2-4AC=0-4*0*-1=0$

   So the equation is parabolic

4. **What type of equations can be soled by using Crank-Nickolson's difference formula?**

   Solution:

   Crank-Nickolson's difference formula is used solve parabolic equations of the form.

$u_{xx} = au_t$

5. For what purpose Bender-Schmidt recurrence relation is used?

Solution:

To solve one dimensional heat equation.

6. Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation $u_{tt} = a^2 u_{xx}$

Solution:

For $\lambda = \frac{1}{a}$, the solution of the difference equation is stable and coincides with the s

olution of the differential equation. For $\lambda > \frac{1}{a}$, the solution is unstable.

For $\lambda < \frac{1}{a}$, the solution is stable but not convergent.

7. What is the purpose of Liebmann's process?

Solution:

The purpose of Liebmann's process is to find the solution of the Laplace equation $u_{xx} + u_{yy} = 0$ by iteration.

8. Define a difference quotient.

Solution:

A difference quotient is the quotient obtained by dividing the difference between two values of a function by the difference between two corresponding values of the independent variable.

PART – B

1. **Solve the Poisson's equation** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-10(x^2+y^2+10)$ **over the square with sides *x=0=y, x=3=y* with *u=0* on the boundary and mesh length is 1.**

2. **Solve the Poisson equation** $u_{xx}+u_{yy}=-81xy, 0<x<1, 0<y<1$ **given that *u(0,y)=0, u(x,0)=0, u(1,y)=100, u(x,1)=100* and *h*=1/3.**

3. **Solve** $u_{xx}+u_{yy}=0$**,** $0\le x,\ y\le 1$ **with *u(0,y)=10=u(1,y)* and *u(x,0)=20=u(x,1)*. Take h=0.25 and apply Liebmann method to 3 decimal accuracy.**

4. **Solve** $y_{tt}=y_{xx}$ **upto t=0.5 with a spacing of 0.1 subject to y(0,t)=0, y(1,t)=0, y_t(x,0)=0 and y(x,0)=10+x(1-x)**

5. **Solve** $32u_t=u_{xx},\ 0<x<1,\ t>0,\ \ u(x,0)=0,\ u(0,t)=0,\ \ u(1,t)=t$ **choose h=0.25**

6. **Using Crank-Nickolson's Implicit scheme, Solve**
   $16u_t=u_{xx},\ 0<x<1,\ t>0,$ **given that u(x,0)=0, u(0,t)=0, u(1,t)=100t**

7. **Approximate the solution to the following elliptic partial differential equation**

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=e^{xy}(x^2+y^2), 0<x<1, 0<y<1, u(0,y)=1, u(1,y)=e^y, 0\le y\le 1 \,\&\, u(x,0)=1, u(x,1)=e^x,$$

$$0\le x\le 1,\ \ using\ \ h=k=\frac{1}{3}$$